

APPLICATION OF THE THEORY OF MECHANISMS TO SOLVE PROBLEMS FROM GEOMETRY

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Abstract: This paper deals with the process by which pure geometric problems, such as determining the position of the triangle or line whose points should lie on circles or straight lines, or that edges of the triangle tangle circles or similar. By moving from the field of geometry to the field of the theory of mechanisms, it is possible to relatively simply describe problem with the system of nonlinear equations to reduce on the problem of finding zeros of the polynomials of a certain degree. Problem of solving a system of nonlinear equations reduces to the problem of linear algebra, which, from the aspect of numerical analysis, is a significant advantage. The paper deals with the solution of a concrete problem.

Keywords: geometric problem, planar linkage mechanisms, mechanisms analysis

1. Introduction

The object of this paper is to solve some pure geometric problems by transforming the same into the problem of the analysis of the position of related kinematic structures, of which plain linkage mechanisms with rotational and sliding kinematic pairs are assembled. After the transformation has been made the problem of geometric analysis is solved by methods of the analysis of the position of the plain linkage mechanisms. The analysis of the position of the plain linkage mechanisms will be performed on the basis of the algebra of complex numbers. After obtaining the analysis solution of the position of the observed structure it is possible to present the solution of the initially observed geometric problem.

2. Setting and procedure for solving the problem

The following possible problems will be considered:

- Determining the position and number of possible positions that the ABC triangular plate occupies so that the ABC points are located on known circles C_A , C_B , C_C (see Fig 1.) or, instead of some circles, straight lines (see Fig 2.)

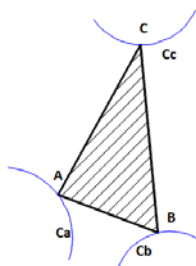


Fig 1.: Paths of points A,B,C are circles

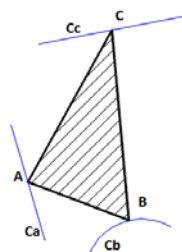


Fig 2.: Paths of points A and C are straight lines, and path of point B is circle

- Determining the position and number of possible positions that the triangular plate ABC occupies so that points A, B occupy on the known circles C_A and C_B , and that the third party of triangle tangles the third

curve C_C (see Fig 3.) or that all three sides of the ABC triangle tangle the circles (see Fig 4.)

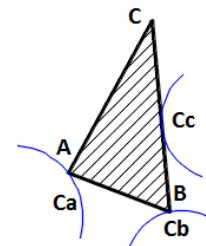


Fig 3.: Paths of points A and B are circles, and line BC tangles circle

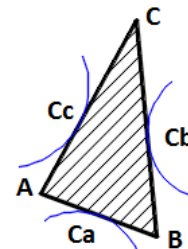


Fig 4.: Lines AB, BC and CA tangles the circles

- Determination of possible positions that line ABC takes along, so for example, points A, B, C of the line lie on circles (see Fig 5.) or, for example, the points A and B of the line lie on the straight lines, and line tangles known circle C_C (see Fig 6.).

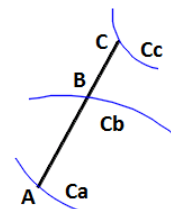


Fig 5.: Paths of points A, B and C are circles

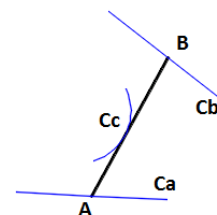


Fig 6.: Paths of points A and B are straight lines, and line AB tangles the circle

All of these problems will be solved forming of appropriate structures with rotational and sliding kinematical pairs. This will enable us to use methods of analysis of the mentioned structures, ie, the methods of analysis of plain linkage mechanisms to solve these problems.

Mechanisms that correspond to these problems look like on the Figure 7.

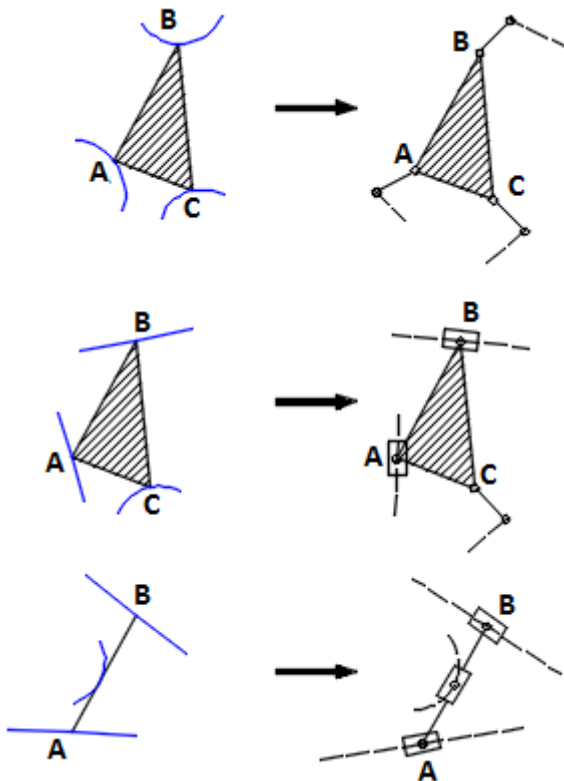


Fig 7. : Transformation of geometric problems into mechanisms

3. Solution of the problem

A concrete problem will be considered. Possible positions of the line AB (see Fig 8) should be found, so that points A and B are respectively located on straight lines C_A and C_B , and line AB passes through the point C (0,0) where $\overline{AB} = 3.0$, and point D has coordinates (-0.5 and 2)

The geometric problem shown in the Figure 8. is joined by the structure, i.e. a mechanism, with rotational and sliding kinematic pairs shown in the Figure 9. [1],[2],[3].

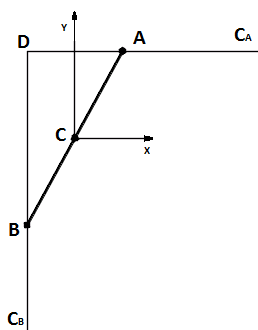


Fig 8.: Geometric problem

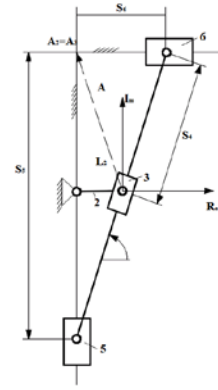


Fig 9.: Mechanism that corresponds to the problem from Fig 8.

$$L_1 e^{i(\varphi - \pi/2)} + S' e^{i(\theta)} + a_k e^{i(\varphi - \pi/2)} = A_k + L_k e^{i(\alpha_k - \pi/2)} + S_k e^{i\alpha_k} = 0$$

$$t = e^{i\varphi}$$

$$u = S'$$

$$-L_1 i t + u t - a_k i t = A_k + S_k i e^{i\alpha_k} - i L_k e^{i\alpha_k} / e^{-i\alpha_k}$$

$$t[-(L_1 + a_k)i e^{i\alpha_k} + u e^{-i\alpha_k}] = A_k e^{-i\alpha_k} + L_k i + S_k \dots (*)$$

$$H = -(L_1 + a_k)i e^{i\alpha_k}$$

$$F = \text{Im}(A_k e^{-i\alpha_k}) - L_k$$

$$\text{Im}(*): \text{Im}(t) \text{Re}(H + u \cos \alpha_k) + \text{Re}(t) (\text{Im}(H) - u \sin \alpha_k) = F$$

$$i \frac{1-t^2}{2t} \text{Re}(H + u \cos \alpha_k) + \frac{1+t^2}{2t} (\text{Im}(H) - u \sin \alpha_k) = F$$

$$t^2 + \frac{-2F}{\text{Im}(H) - \text{Re}(H)i - (\cos \alpha_k i + \sin \alpha_k)u} t + \frac{\text{Re}(H)i + \text{Im}(H) + (\cos \alpha_k i + \sin \alpha_k)u}{\text{Im}(H) - \text{Re}(H)i - (\cos \alpha_k i + \sin \alpha_k)u} = 0$$

$$\text{gdje je: } 2B = \frac{-2F}{\text{Im}(H) - \text{Re}(H)i - (\cos \alpha_2 i + \sin \alpha_2)u}; C = \frac{\text{Re}(H)i + \text{Im}(H) + (\cos \alpha_k i + \sin \alpha_k)u}{\text{Im}(H) - \text{Re}(H)i - (\cos \alpha_k i + \sin \alpha_k)u}$$

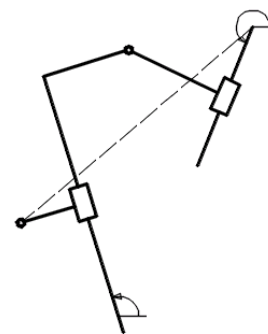


Fig. 10: Structure R-S-R-S

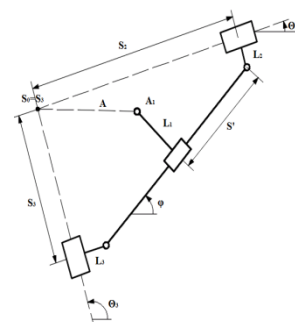


Fig. 11. : Structure R-S-R-S with symbols

In its structure Assur's structural group of III class (see Fig 9.) has three rotational and three sliding kinematic pairs of which two are external, and one is internal [1]. General shape of this structure is given on the Figure 11.

It can be concluded that observed structure consists of two chains of the same structure: R-S-R-S and R-S-R-S. For the chain R-S-R-S for $k=2$ it is: $L_1=0$; $L_2=0$; $\theta_2=0$ so it has shape shown in Figure 12:

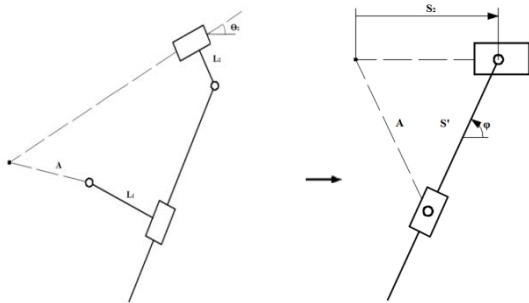


Fig 12.: First structure R-S-R-S

For the chain R-S-R-S for $k=3$ it is: $L_1=0$; $L_3=0$; $\theta_3=-\pi/2$; $a_3=3$ so it has shape shown in Figure 13:

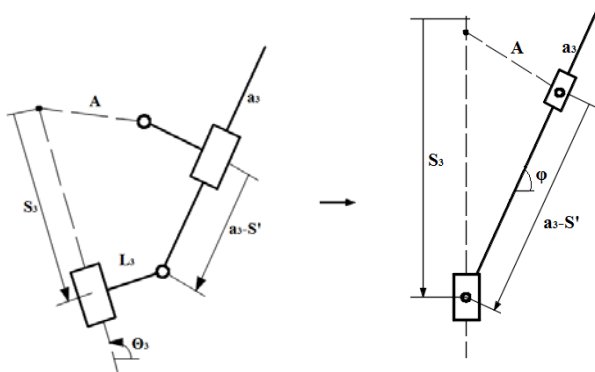


Fig 13.: Second structure R-S-R-S

For each of the chains (see Fig 12. and Fig 13.) equations of the following shape can be written:

$$t^2 + 2B_k t + C_k = 0; k = 2, 3$$

ie. equations can be written:

$$t^2 + 2B_2 t + C_2 = 0$$

$$t^2 + 2B_3 t + C_3 = 0$$

where: $t = e^{i\varphi}$; $u = S'$;

$$A = -0,5 \cdot i \cdot 1,2; H_2 = -(L_1 + a_2)e^{i\alpha_2} = 0 + i0; F_2 = 1,2; 2B_2 = \frac{-2 \cdot 1,2}{-u \cdot i};$$

$$c_2 = \frac{u}{-u} = -1; H_3 = 0 + i0; F = -0,5; 2B_3 = \frac{-2(-0,5)}{(3-u)i} = \frac{1}{(3-u)i}; c_3 = \frac{u}{u} = 1$$

so previous system of the equations has shape as:

$$t^2 + \frac{2,4}{u \cdot i} t - 1 = 0;$$

$$t^2 + \frac{1}{(3-u)i} t + 1 = 0;$$

It is not hard to show that system of previously given two square equations, after elimination of t , can be written as polynomial of fourth degree with real variable u and real coefficients

$$u^4 - 6,0 u^3 + 7,31 u^2 + 8,64 u - 12,96 = 0$$

4. Results and conclusion

With determining u geometry of the structure is defined, where all possible solutions of the equation are considered, which would not be as simple as if the given system of four equations as in the beginning of the example was to solve[3].

Four roots of the polynomial are:

$$u = (s)_1 = 1.252; t = e^{\theta_{41}} = 0,286 + 0,958i; \theta_{41} = 73.375$$

$$u = (s)_2 = 2.425; t = e^{\theta_{42}} = 0,869 + 0,495i; \theta_{42} = 29.664$$

$$u = (s)_3 = 3.532; t = e^{\theta_{43}} = -0,94 + 0,34i; \theta_{43} = -19.863$$

$$u = (s)_4 = -1.209; t = e^{\theta_{44}} = 0,119 - 0,993i; \theta_{44} = -83.177$$

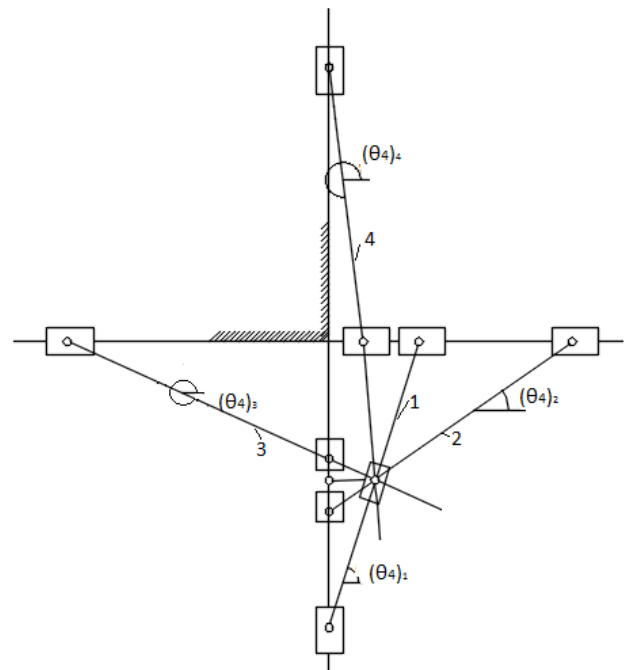


Fig. 14. :Solution of the mechanism positions

Solution of the problem of mechanism analysis is given on the Figure 14., and solution of the starting geometric problem is given on the Figure 15.

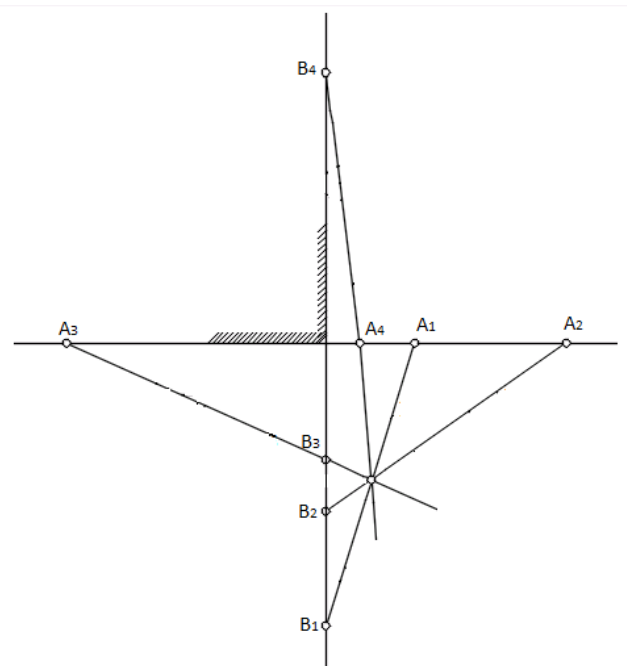


Fig. 15. :Solution of the starting geometric problem

First, a geometric problem is defined, that is: point A and point B, and line AB must lie on a horizontal and vertical straight line, and line AB should tangent the circle (whose radius is zero). Secondly, to the geometric problem corresponds the mechanism consisting of two R-S-R-S structures. Thirdly, the analysis of the observed structure, ie the mechanism, based on complex algebra was performed. The solutions obtained define the 4 positions of the mechanism. Fourth, by returning to the starting geometric problem, 4 positions of the line AB are defined such that point A is on the horizontal line, point B on a vertical line, and the AB line touches the circle with the center at the point C of the radius equal to zero, that is, line AB passes through point C.

5. Literature

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- [3] G. ĆULAFIĆ: *Analitička geometrijska analiza polužnih mehanizama više klase*, XXI Jugoslovenski kongres teorijske i primijenjene mehanike, Niš, 1995